# Discrete lattice model used to describe the fracture process of concrete

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# INTRODUCTION

The fracture process being a major case of damage in quasi-brittle materials under mechanical loading is highly complex due to the heterogeneous structure of materials over many different length scales changing in concrete from the nanometer (hydrated cement) to the milimeter (aggregate particles). The fracture patterns consist of a main crack with various branches, secondary cracks and micro-cracks. The cracks contribute to a significant degradation of the material strength. They can be modelled with continuum models (e.g. fracture, elasto-plastic, damage and coupled plastic-damage approaches) enhanced by a characteristic length of the micro-structure (to restore a well-posed problem and remedy mesh dependency) and with discrete models (e.g. particle and lattice approaches). A realistic description of the fracture process is of a major importance to ensure safety of the structure and to optimise the material behaviour.

In a lattice model [2]-[4], [6] material is discretized as a lattice of beam elements transferring forces. Fracture is simulated by performing a linear elastic analysis under loading and removing a beam element that exceeds tensile strength. The displacement vector is calculated similarly as in the conventional FEM (by multiplication of the inverse global stiffness matrix with the load vector). The heterogeneity of the material can be taken into account by assigning different strengths to beams (using a Gaussian or Weibull distribution), by assuming random dimensions of beams and random geometry of the lattice mesh or by a mapping of different material properties to beams corresponding to the cement matrix, aggregate and interface zones. For the distribution of aggregate grains, a Fuller curve is usually chosen.

The advantage of this approach is simplicity and a direct insight in the fracture process on the level of the micro-structure. The disadvantages are following: the results depend on the beam size and direction of loading, the response of the material is too brittle (due to the assumed brittleness of single beams), the compressed beam elements overlap and an extreme computational effort on the structure level is needed.

The intention of simulations presented in this paper was to present the potential of a lattice model to model the fracture process in a concrete-like material during different two-dimensional processes of loading.

#### DESCRIPTION OF THE MODEL

The lattice model proposed in the paper has a geometric type [2], [5]. The beam node displacements were calculated successively on the basis of geometric changes of other nodes (thus, the global stiffness matrix was not built). The beams possessed a longitudinal (tensilecompressive) stiffness described by the parameter  $k_l$  and bending stiffness described by the parameter  $k_b$ . The beam deformations were described by a curvature angle, elongation and shear angle. The arbitrary displacements of beam nodes were obtained from the weighted sum of longitudinal node displacements multiplied with the longitudinal stiffness parameter and perpendicular node displacements (due to rotations) multiplied with the bending stiffness parameter. The distribution of beams was assumed to be completely random. First, a square grid of boxes was created. In each box of the grid, one point was selected at random. Next, the points inside of boxes were connected with neighboring points to create a mesh of beams. The beams were removed from the lattice if the critical tensile strain  $\varepsilon_{min}$ , critical compressive strain  $\varepsilon_{max}$ , critical bending angle  $\varphi_b$  (calculated from the sum of node rotations) or critical shearing angle  $\varphi_s$  (calculated from the subtraction of node rotations) were exceeded. The same properties were assumed for all beams (i.e. presence of aggregate and cement paste has not been considered yet). 10000 beam elements were used in numerical simulations. In the first step, one assumed the following parameters:  $\varepsilon_{min}=0.02\%$ ,  $\varepsilon_{max}=0.20\%$ ,  $\varphi_b=0.015$  rad and  $\varphi_{s}$ =0.015 rad. The lattice model computations are purely static, and no energy check is made during calculation process.

## RESULTS

Fig.1 presents the change of the Poisson's ratio against the relationship between the longitudinal stiffness  $k_l$  to the bending stiffness parameter  $k_b$ . For the ratio of  $k_l/k_b=0.75$ , the Poisson's ratio for concrete was obtained (v=0.2). If the ratio  $k_l/k_b>1$ , the Poisson's ratio became negative.

Figures 3 and 4 show the propagation of cracks in a specimen during extension for different loading states. The evolution of the mean vertical stress on the top edge versus the specimen deformation during extension is presented in Fig.2.

The obtained crack patterns are qualitatively in agreement with model experiments [3]. The presence of an initial imperfection significantly influences the crack pattern. The vertical resultant force on the top edge grows very quickly, reaches a maximum and shows a pronounced softening. However, the stress-strain curve is too brittle as compared to experiments [3].

Figures 5-8 show the distribution of cracks in a concrete-like specimen subject to uniaxial compression, shear, combined shear-extension and bending. The cracks are straight or curvilinear. The obtained crack patterns are qualitatively in agreement with laboratory experiments [3].



Fig.1. Influence of the relationship between the longitudinal stiffness to the bending stiffness on the Poisson's ratio



Fig.2: Load-displacement curve for a specimen subject to uniaxial extension



Fig.3: Crack propagation in a specimen with smooth edges subject to uniaxial extension (without an initial imperfection)



Fig.4: Crack propagation in a specimen with smooth edges subject to uniaxial extension (with an initial imperfection)



Fig.5: Formation of cracks in a specimen subject to uniaxial compression:

a) smooth edges, b) very rough edges



Fig 6: Formation of cracks in a specimen subject to simple shearing



Fig.7. Formation of cracks in a specimen subject to combined shear-extension



Fig.8. Formation of cracks in a specimen subject to bending

# CONCLUSIONS

The lattice model is a simple approach to the fracture behavior in quasi-brittle materials but very helpful in studying the formation of cracks. Owing to it, novel (stronger and better) materials can be developed. The obtained results are qualitatively in agreement with experimental ones.

The calculations with a lattice model will be continued. First the model will be extended into 3D. Energy conservation check will be added, for dynamic symulations. The non-linear fracture law with softening will be used. The deformations will be non-locally calculated to increase the material ductility. The different properties of aggregate, cement paste and interface zones will be distinguished in specimens. To prevent the overlapping of compressed

beams, special boundary elements will be introduced. The calculations will also be performed with reinforced concrete elements.

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