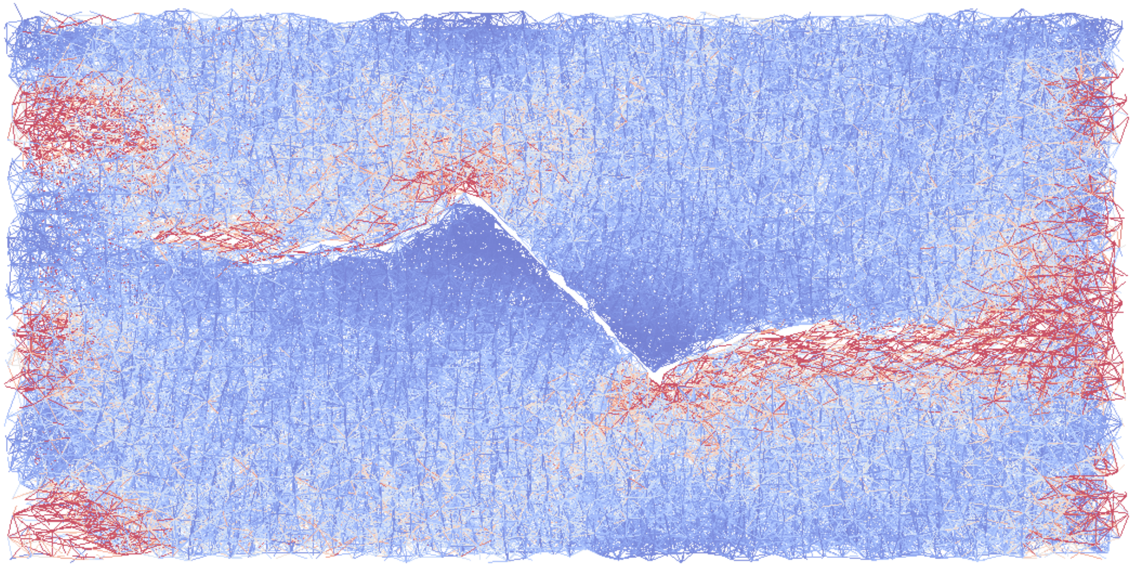

Discrete Element Based Hydraulic Fracture Model

Test Case 3: Single fracture in homogenous poroelastic, thermo elastic media

(a) Newtonian fluid without proppant in a poroelastic media

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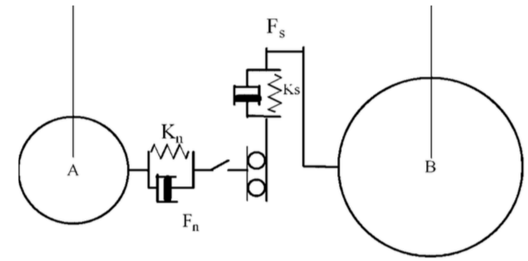
Discrete Element Method

1. Discrete element tracking (Yade): Newton's 2nd Law: $\ddot{u} = F/m$



2. Determination of forces (particle interactions):

$$k_n = \frac{E_1 r_1 E_2 r_2}{E_1 r + E_2 r_2} \;\&\; k_s = \upsilon k_n \mid F_n = k_n \Delta D \;\&\; F_s = F_{s,prev} + k_s \Delta u_S$$

3. Failure criteria (Scholtes and Donze 2012):

$$F_{n_{max}} = -tA_{int} \;\&\; F_{s_{\max}} = F_n \tan\varphi + cA_{int}$$

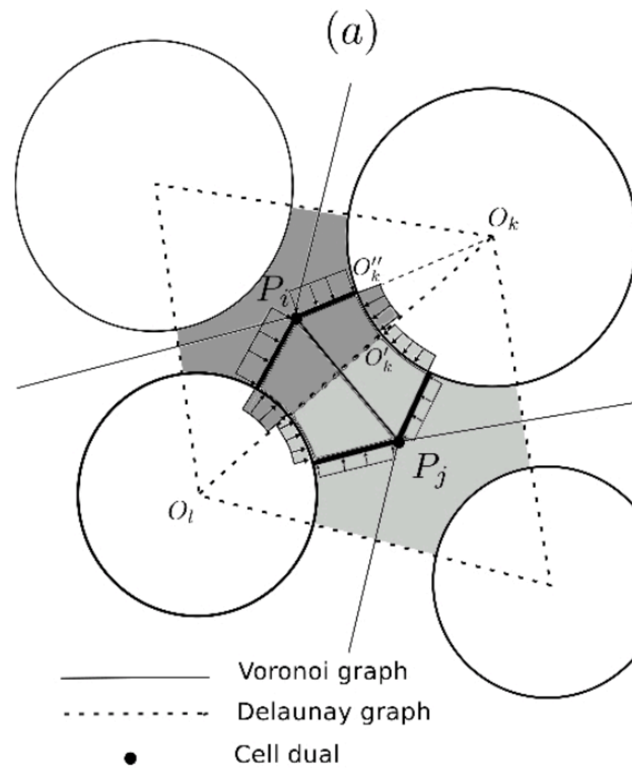
4. Fluid coupling (Yade, Chareyre et al. 2012):

$$[\boldsymbol{G}]\{\boldsymbol{P}\} = [\boldsymbol{E}]\{\dot{\boldsymbol{X}}\} + \boldsymbol{Q_q}$$

5. Triangulation created using particles as nodes

6. Conductance governed by Poiseuille's law (Papachristos, 2017):

non fractured, $k = \alpha \frac{A_{ij} {R_{ij}^{h}}^2}{\mu}$ & fractured, $k = \frac{h^3}{12\mu}$



7. Pressure and viscous forces on particles:

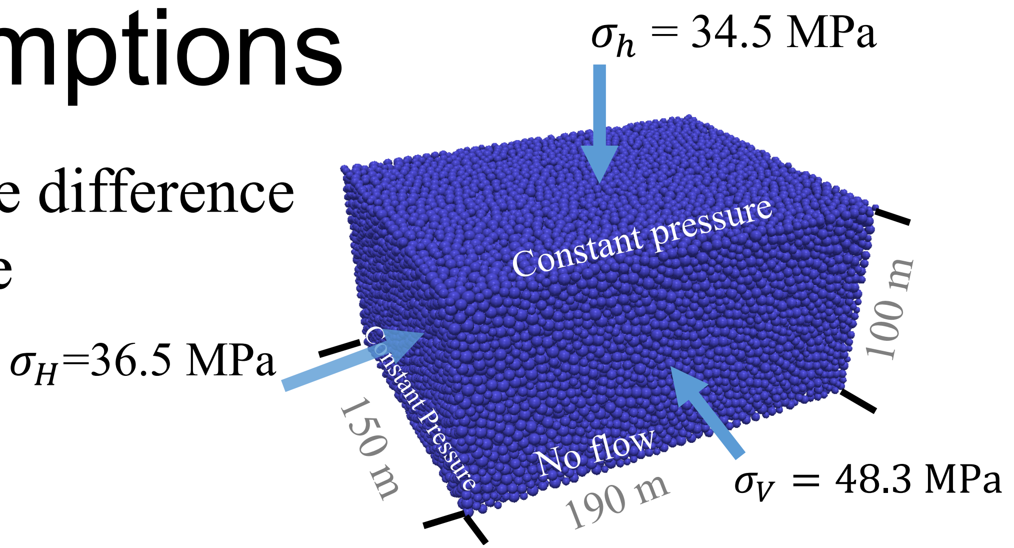
$$F_p = A_p(p_i - p_j)\boldsymbol{n} \;\&\; F_{v_{total}} = A_f(p_i - p_j)\boldsymbol{n}$$

$$F_{v,p} = F_{v,total}\,\gamma \;\&\; \gamma = A_p/A_{total}$$

A_f=pore throat cross section, p = pore pressure $\boldsymbol{G}$ = conductance matrix, $\boldsymbol{E}\dot{\boldsymbol{X}}$ = rate of volume change, $\boldsymbol{P}$ = pore pressures, $\boldsymbol{Q_q}$= source term, F = force m = mass, $\ddot{u}$ = acceleration, μ = dynamic viscosity, υ = microscopic Poisson's ratio, k = stiffness t = tensile strength, A_{int} = interaction area, c = cohesion, k = permeability, R_{ij}^{h} = hydraulic radius, h = separation distance, ΔD = particle overlap, A_p = area of particle on pore, $F_{v/p}$ = viscous/pressure force, φ = friction angle, α = perm. coeff.

Numerical Methods and Assumptions

- Particle position - explicit finite difference
- Fluid flow – pore finite volume

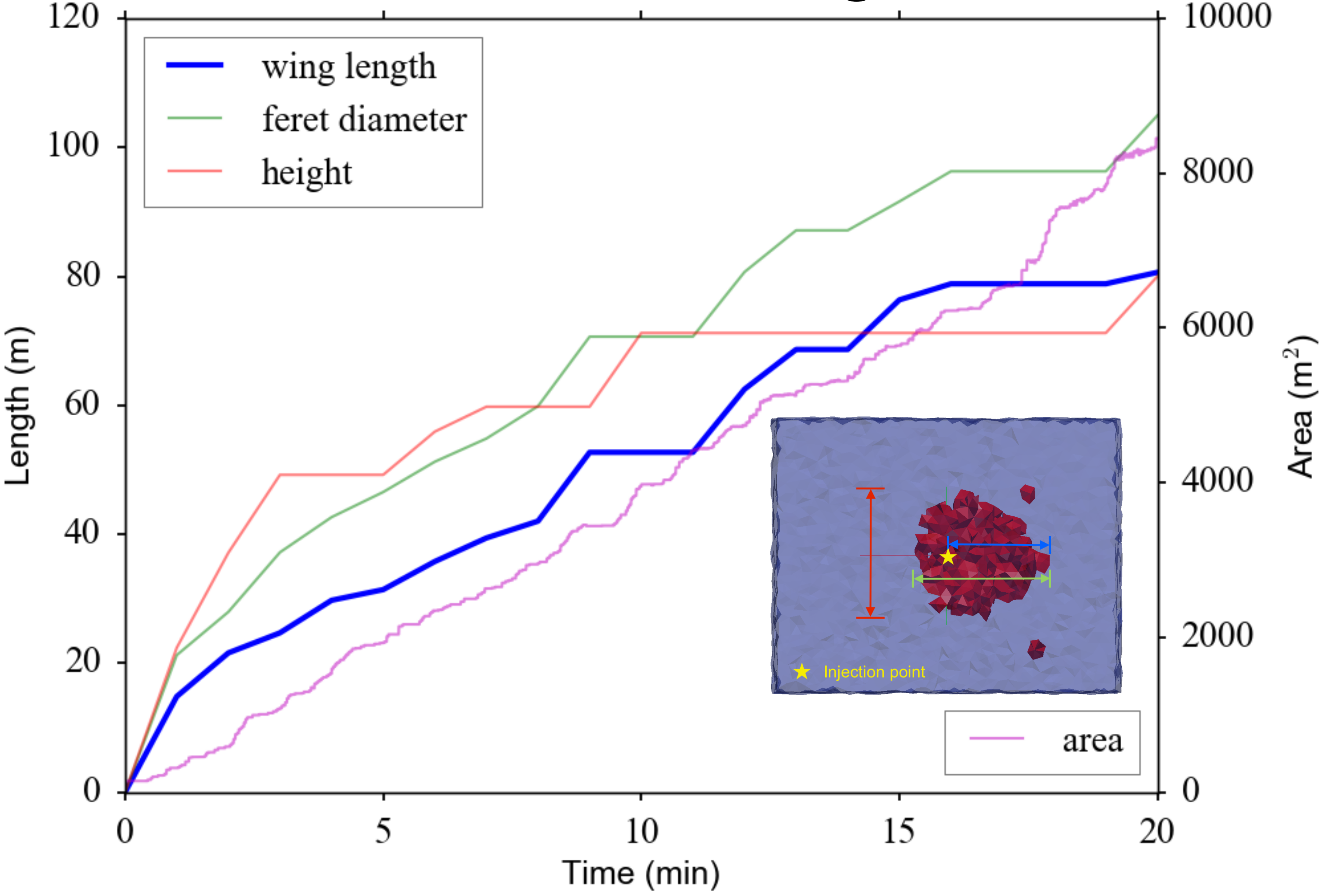


Model Assumptions:

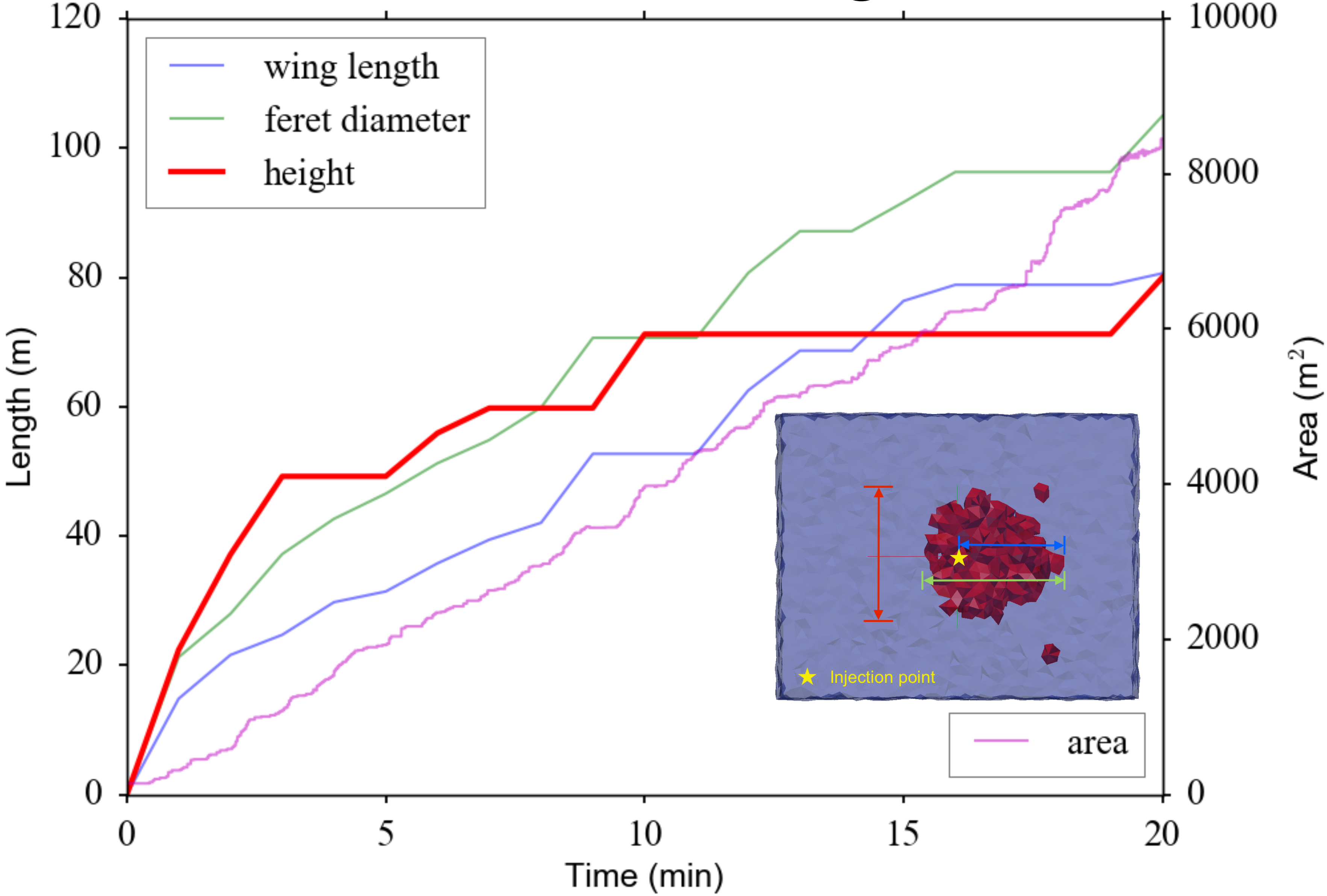
- Matrix permeability – Poiseuille's law
- Fracture permeability – parallel plate approximation
- Mohr-coulomb failure criteria based on particle size
- Broken bonds contain residual fracture width
- Calibrated micro-parameters yield emergent behaviors according to specified macro parameters
- 10 cm perforation depth
- Constant pressure and stress boundary conditions
- No vertical flow out of layer of interest

Micro parameter	Value (DEM)	Description
E_i	32 (GPa)	Young's modulus
k_s/k_n	0.05	Stiffness ratio
ϕ	25^o	friction angle
c	15 MPa	cohesion
t	2.3 MPa	tensile strength
γ_{int}	1.329	interaction range
r	1.5 m $\pm$ 0.25	particle radii
ρ	5000 kg/m^3	particle density
n	0.38	pack porosity
P_p	27 MPa	reservoir pressure
k_{factor}	9e-16	permeability factor
K_{fluid}	2.2 GPa	fluid bulk modulus
μ	0.001 Pa*s	viscosity
$h_{residual}$	1e-6 m	residual aperture

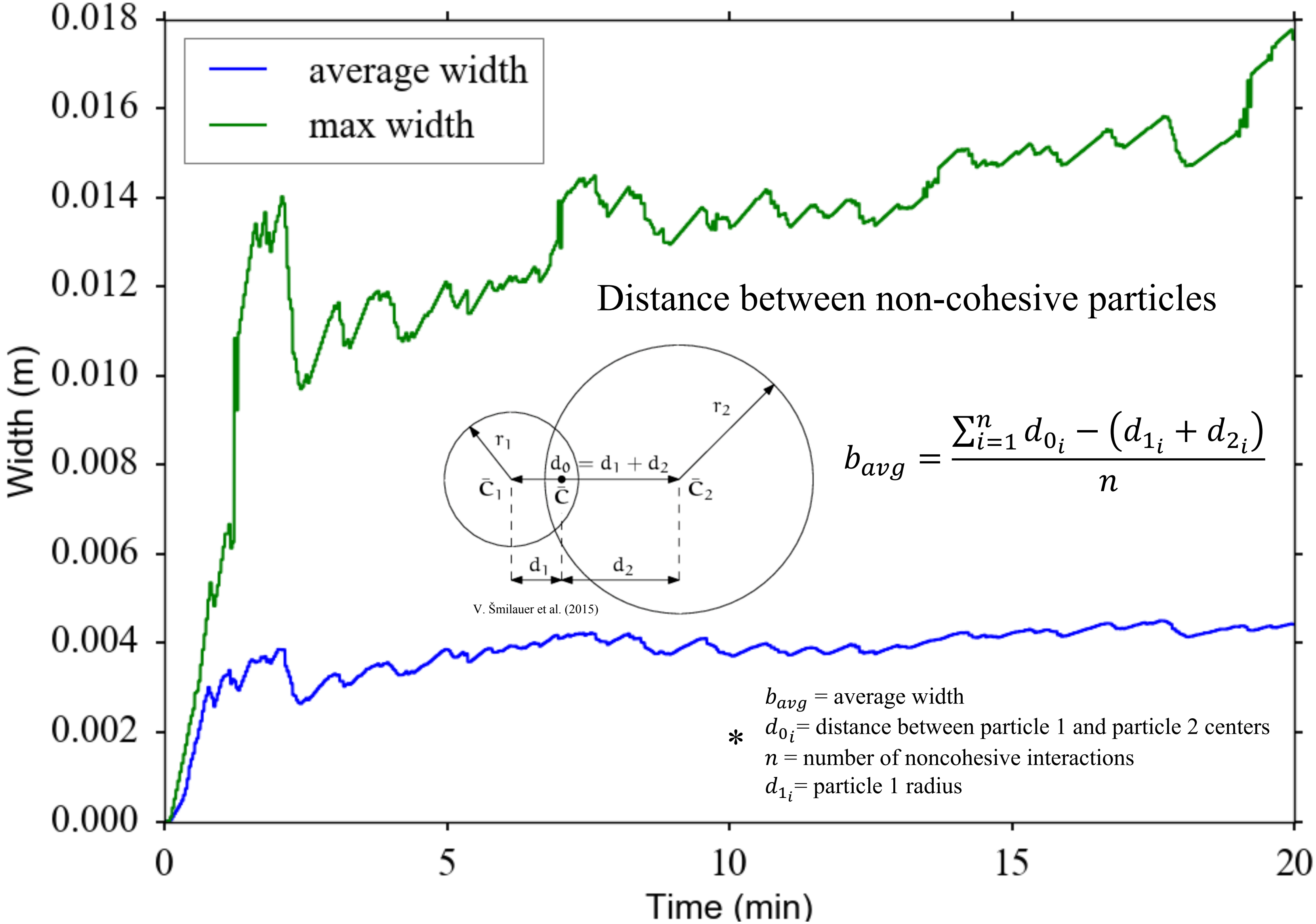
Fracture Length



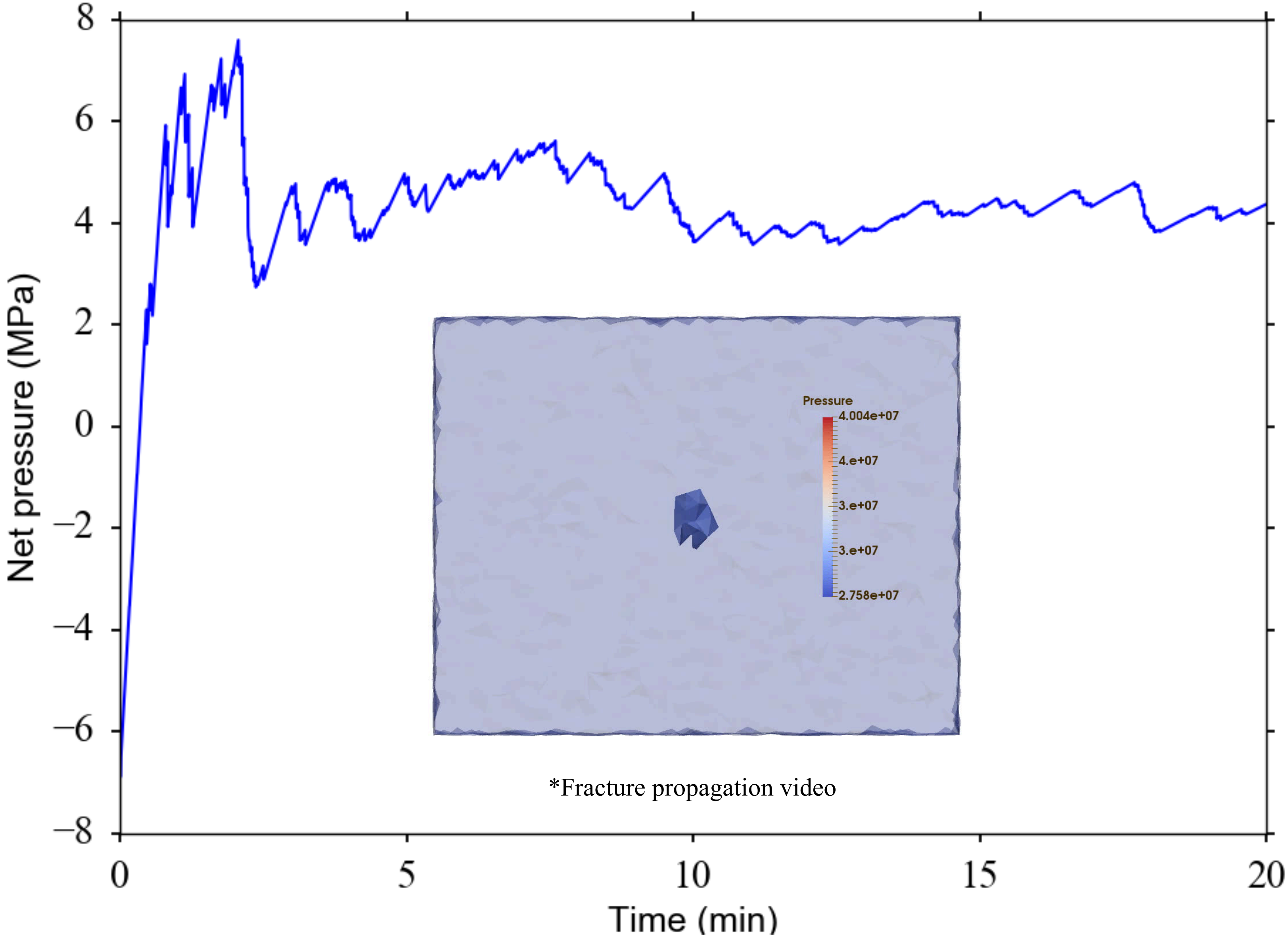
Fracture Height



Fracture Width

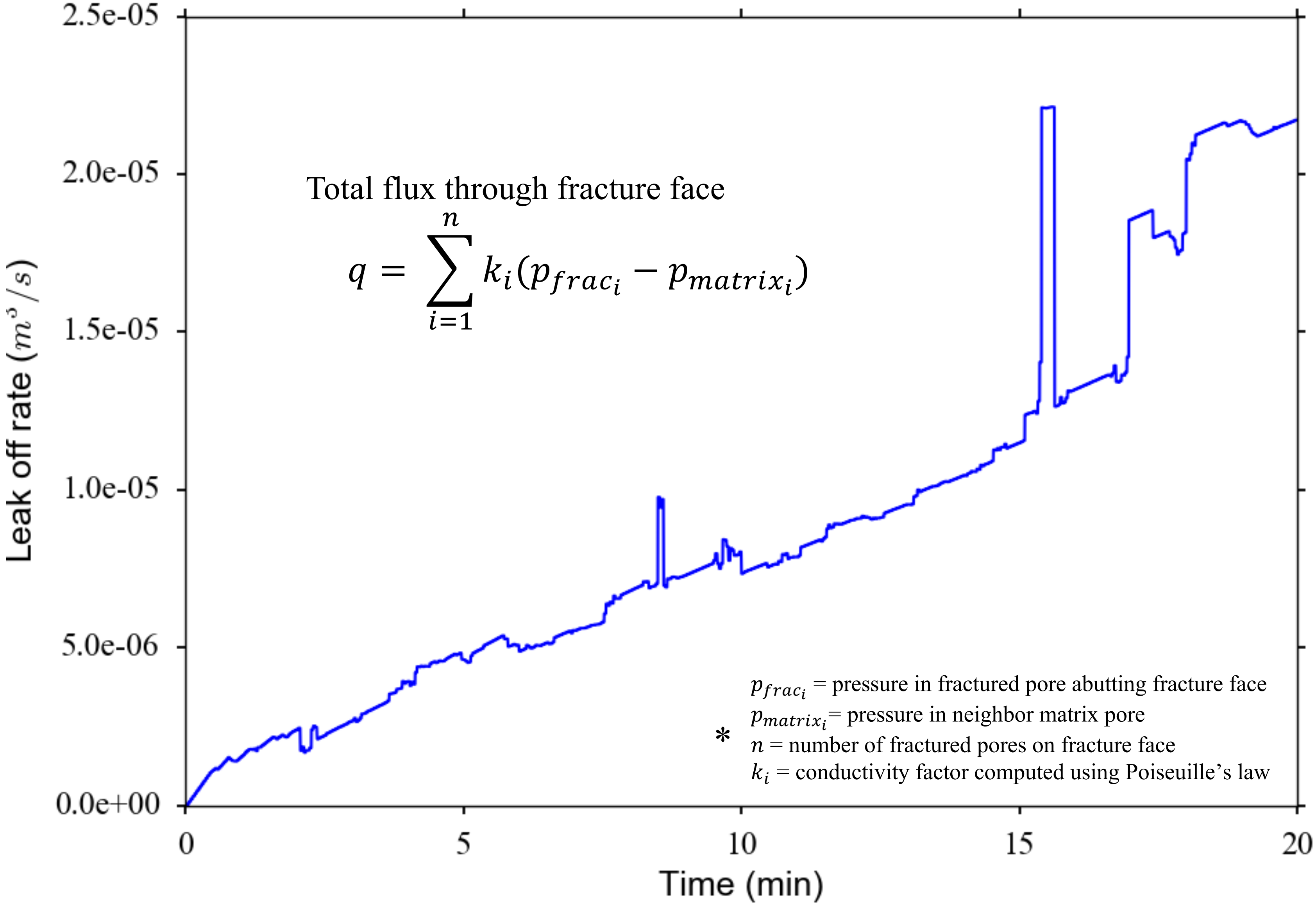


Net Pressure

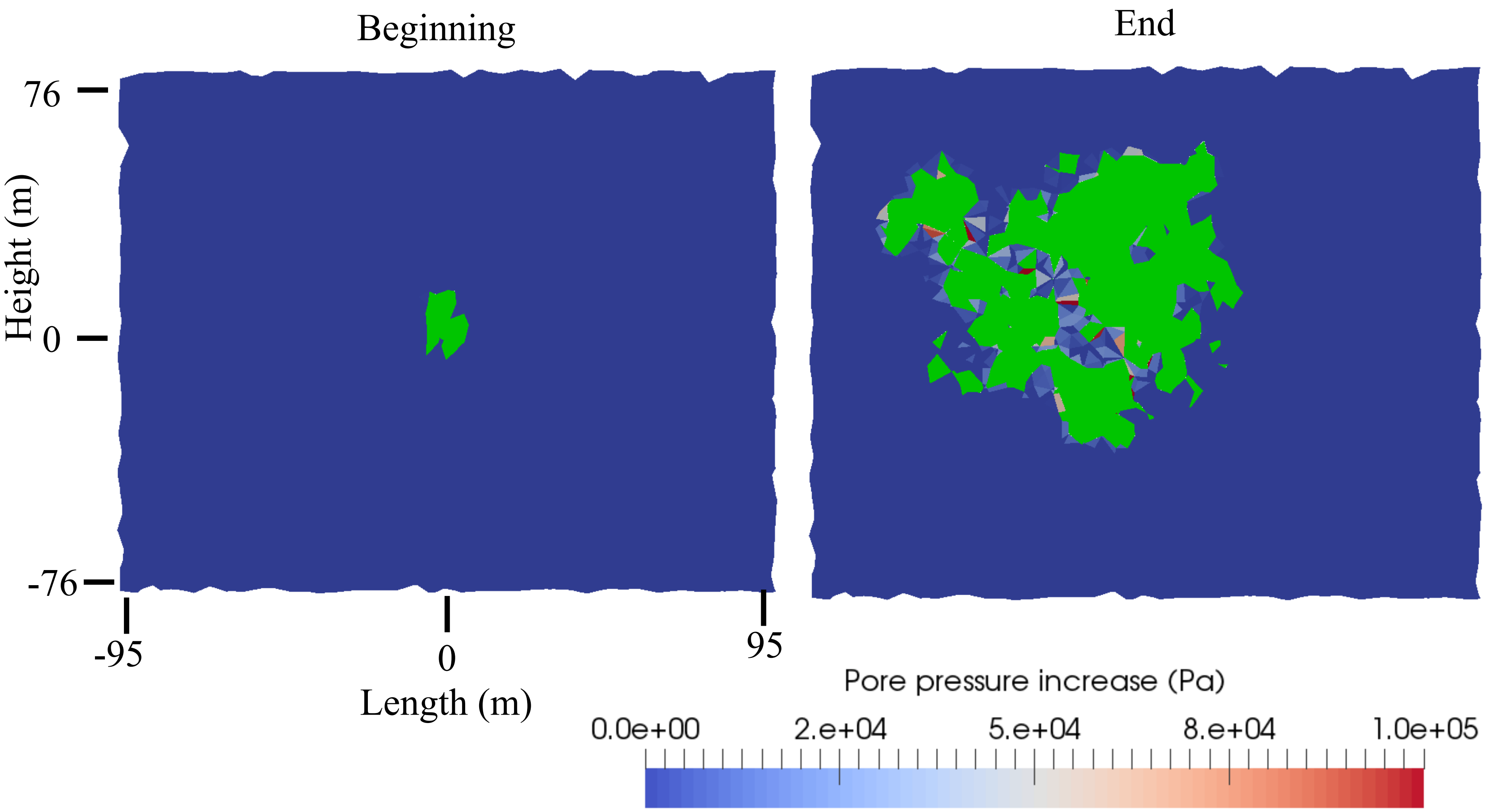


*Fracture propagation video

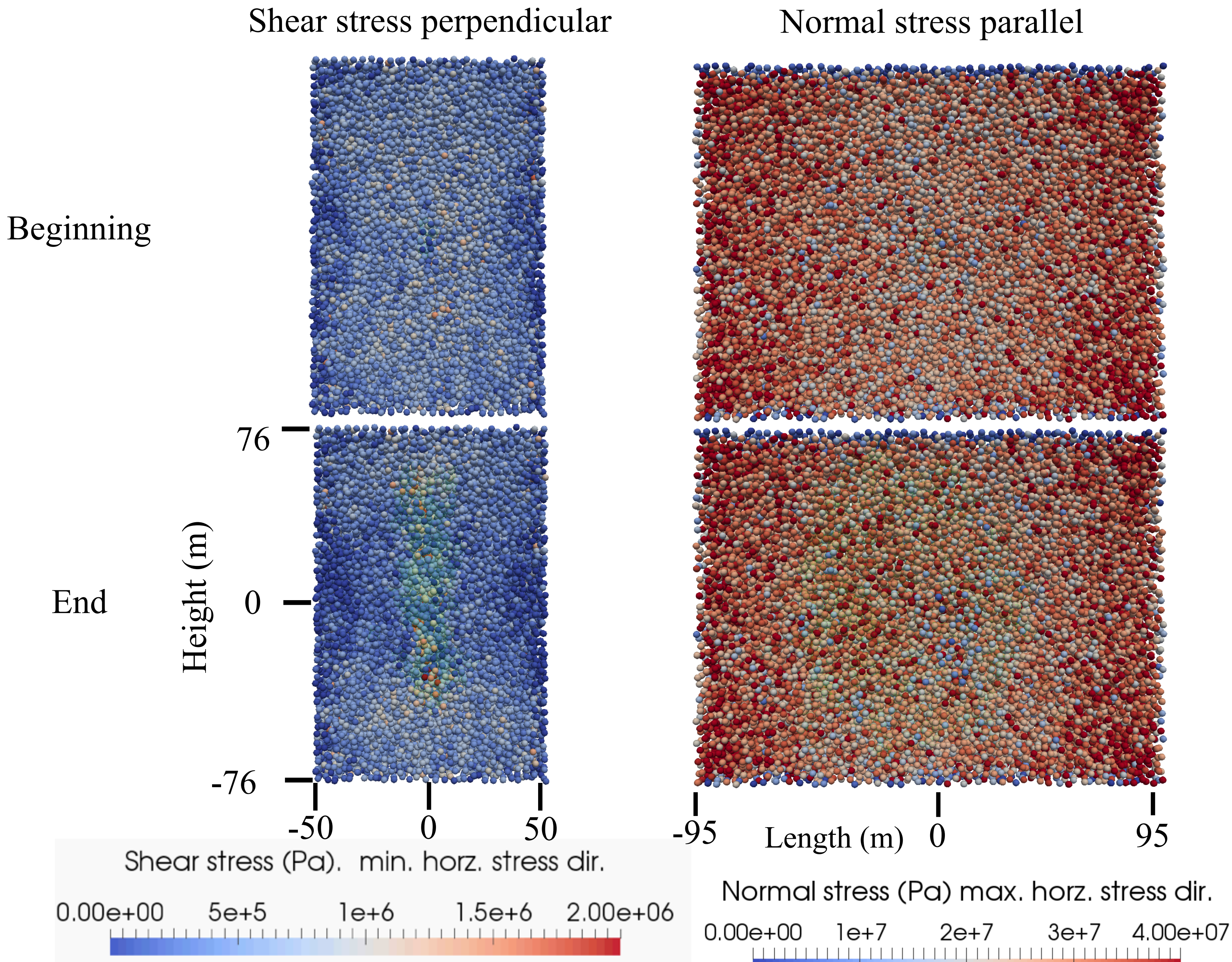
Leak-off Rate



Pore Pressure

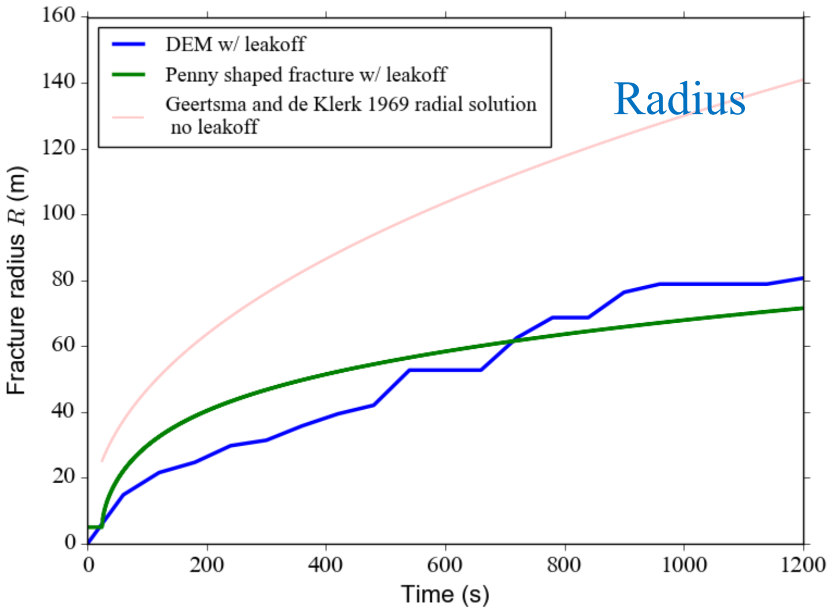


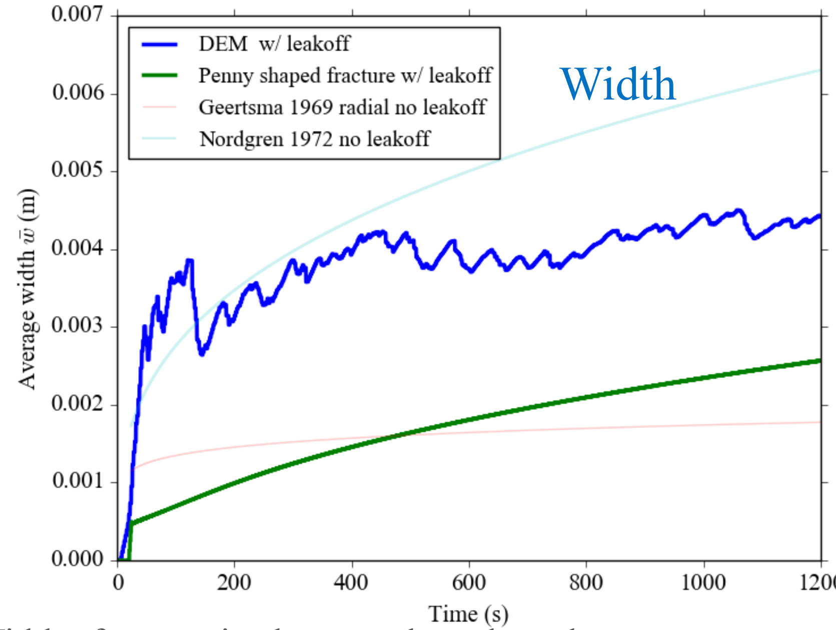
*Fractured cells shown in green

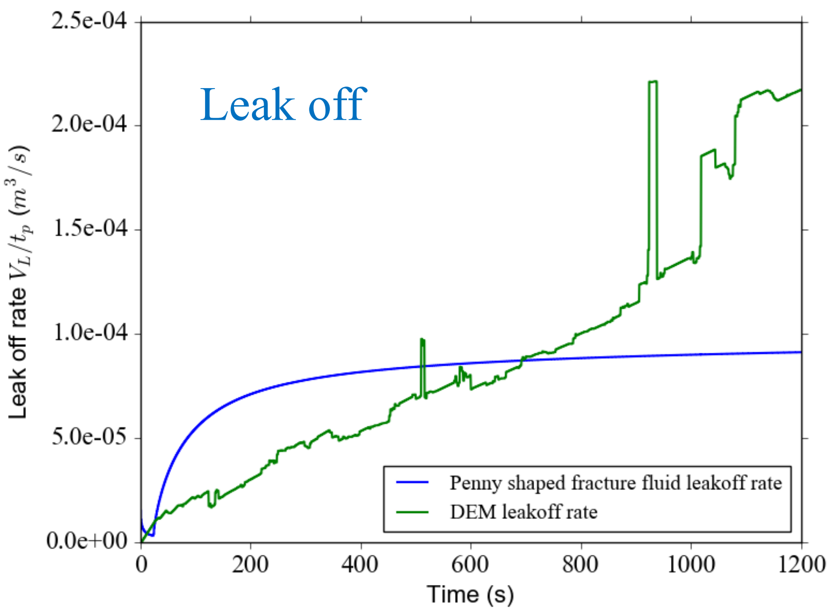


*Fractured cells shown in green

Extra Plots – analytical comparison







Width of pressurized penny shaped crack (Sneddon and Elliot 1946)

$$w(r) = \frac{8 p_{net} R}{\pi E'} \sqrt{1 - (r/R)^2}$$

Conservation of volume:

$$\Delta V_f = q_i \Delta t_p - V_L$$

$$V_L = 2 C_L A_L \sqrt{t} + S_p$$

Propagation pressure (Perkins and Kern 1961):

$$p_{net,c} = \left(\frac{2\pi^3 \gamma_F E'^2}{3V} \right)^{1/5}$$

* V_L=leak off volume, q_i = injection flow rate
w = fracture width, p_{net} = net pressure, C_L=leak off coeff= 1e-5 ft/$\sqrt{min}$
E' = Young's modulus γ_F = fracture energy ,R = fracture radius