Simulations of fracture process in concrete using a 3D lattice model

J. Kozicki, J. Tejchman

Laboratoire 3S – R, Grenoble Universities

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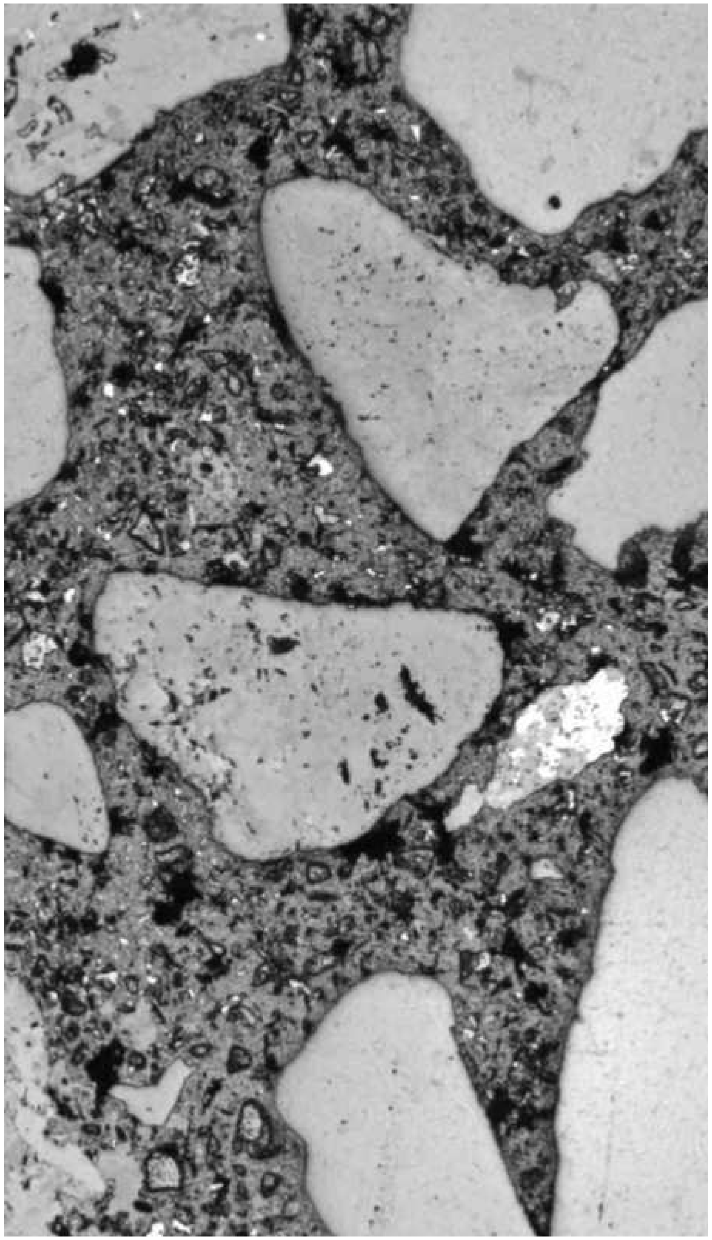
Faculty of Civil and Environmental Engineering,

Gdańsk University of Technology

- Engineering materials such as sand, concrete, rock, ceramics and polymers have common properites:
  - heterogenity
  - anizotropy
  - discrete structure
  - nonlinear behaviour
- Two kinds of numerical models are commonly used:
  - **continuum models** (within fracture, damage, softening plasticity mechanics),
  - **discrete models** (molecular dynamics, discrete element method, lattice models).



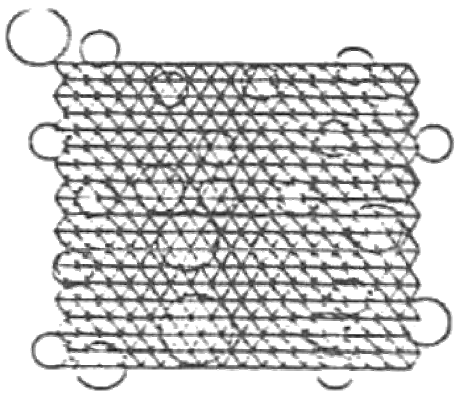
Structure of concrete

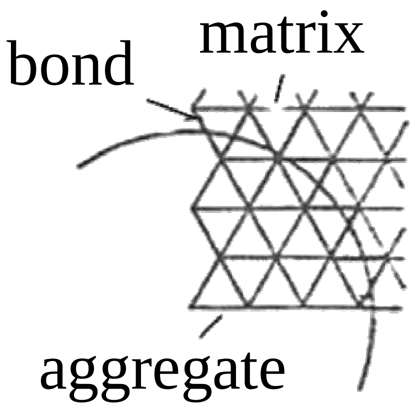
Aim

- The goal is to create a three dimensional discrete lattice model to describe the behavior of quasi-brittle materials.
- Investigate the effect of aggregates and interfacial zones on the 3D fracture process.
- Investigate the size effect on 3D speciemens subject to uniaxial tension

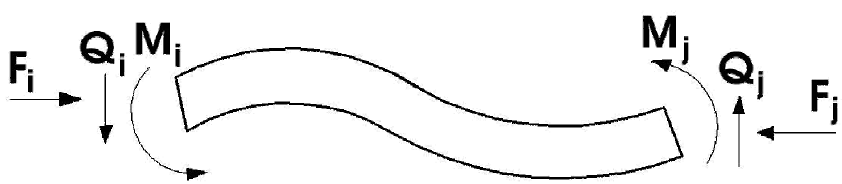
To describe the fracture process in concrete on the scale of cement matrix and aggregates (meso scale) a lattice model was applied.

Original lattice model (Vervuut et al. 1994)

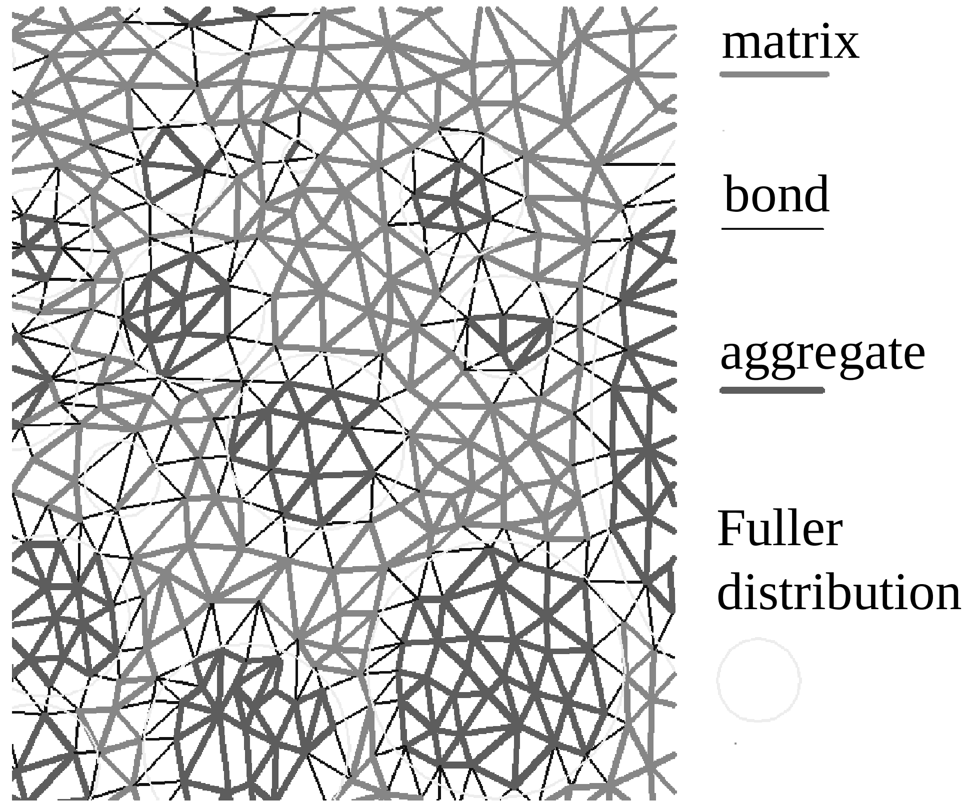




The body is discretized into a mesh of discrete beams.



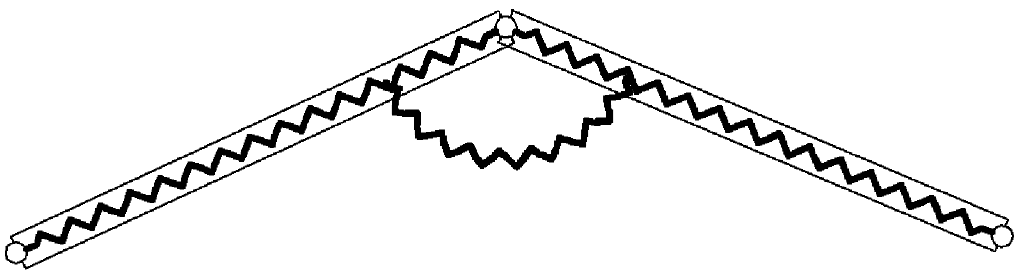
Implicit FEM method is used



Different stiffness and strengths are assigned to various phases.

- Own model – the idea of truss lattice was enchanced by angular stiffnesses that control the rigidity of nodes (bending, shearing and torsion).

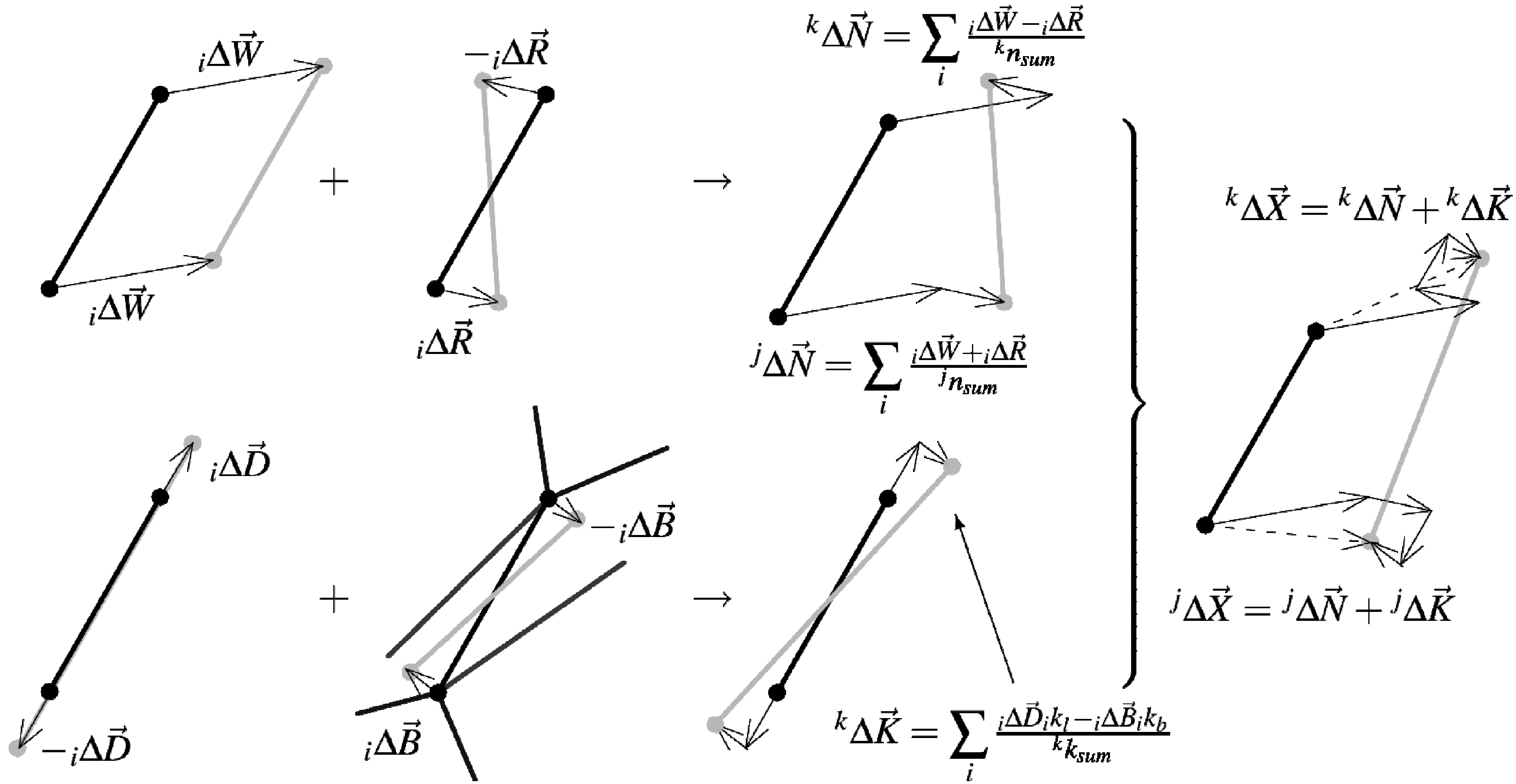
- It is an approach between a truss (nodes not fixed) and a frame (fixed nodes). The rods that connect nodes do not bend, they are connected by angular springs instead.



Connection of rods at the node

- Elements are removed when critical tensile ε_{min} strain is exceeded

Displacement and rotation (2D)



ΔW – movement, ΔR – rotation,

ΔD – change of length due to longitudinal stiffness (k_l),

ΔB – rotation due to bending stiffness (k_b)

Calculating node displacement

$$ {}^{j}\Delta\vec{X} = \sum_i \frac{{}_i\Delta\vec{W} + {}_i\Delta\vec{R}}{{}^{j}n_{sum}} + \frac{\sum_i \frac{1}{{}_id_{init}} \left({}_i\Delta\vec{D}\,{}_ik_l + {}_i\Delta\vec{B}\,{}_ik_b + {}_i\Delta\vec{T}\,{}_ik_t\right)}{\sum_i \frac{1}{{}_id_{init}} \left({}_ik_l + {}_ik_b + {}_ik_t\right)} $$

summation over rod index *'i'*

${}_id_{init}$ – initial length of rod *'i'*

${}^{j}n_{sum}$ – total number of rods connected with the node *'j'*

ΔW – movement,

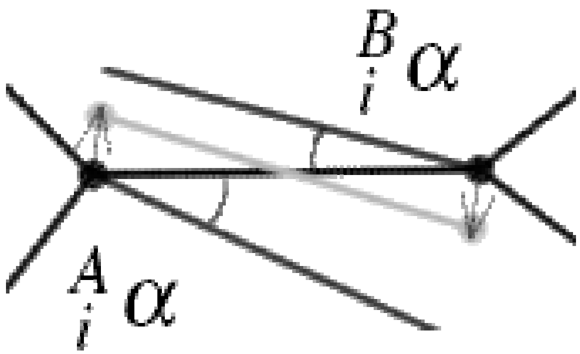
ΔR – rotation,

ΔT – torsional rotation due to torsional stiffness (k_t),

ΔD – change of length due to longitudinal stiffness (k_l),

ΔB – rotation due to bending stiffness (k_b)

Calculation of strain, stress and force



Strain → Stress

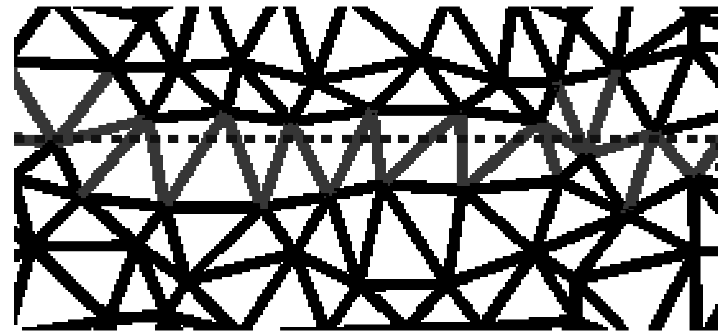
$$_i\varepsilon = \frac{_id - _id_{init}}{_id_{init}} \qquad _i\sigma = {_i\varepsilon} E \, {_ik_l}$$

Shear angle → Shear stress

$$_i\gamma = \frac{{}^A_i\alpha + {}^B_i\alpha}{2} \qquad _i\tau = {_i\gamma} G \, {_ik_b}$$

Bending angle → Cosserat bending stress

$$_i\chi = \frac{{}^A_i\alpha - {}^B_i\alpha}{2} \qquad _im = {_i\chi} G \, {_id^2} \, {_ik_b}$$

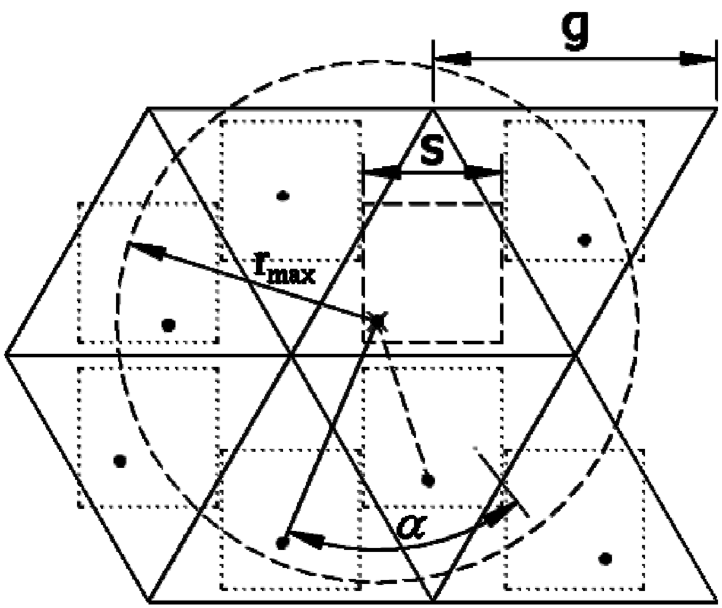


Calculation of section force (ε and γ are projected on normal direction of cross-section plane A)

$$F = A \sum \left(\varepsilon k_l E + \gamma k_b G \right)$$

- E – Young's modulus
- G – Shear modulus
- $_id$ – element length
- $_ik_l$ – longitudinal stiffness
- $_ik_b$ – bending stiffness
- i – element index

Mesh generation method



Mesh generation parameters:

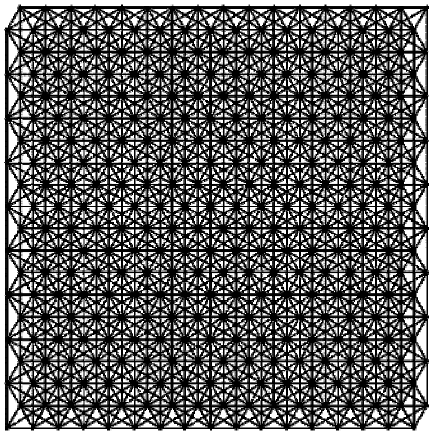
g - cell size [m]

r_{max} - max beam length [m]

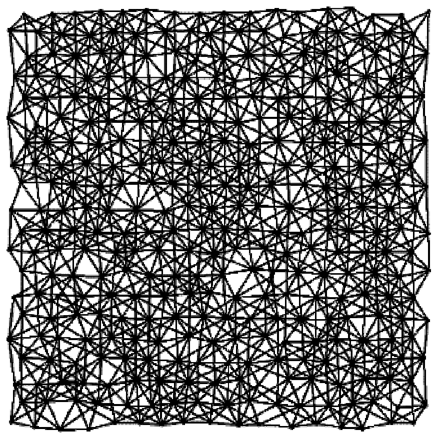
a - min angle between beams [rad]

s - mesh irregularity [m]

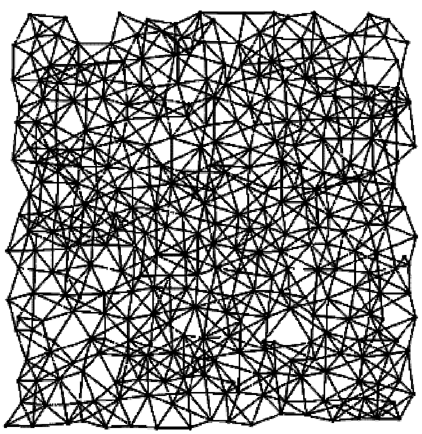
a) s = 0
b) s = 0.3
c) s = 0.6
d) s = 0.6, Delaunay



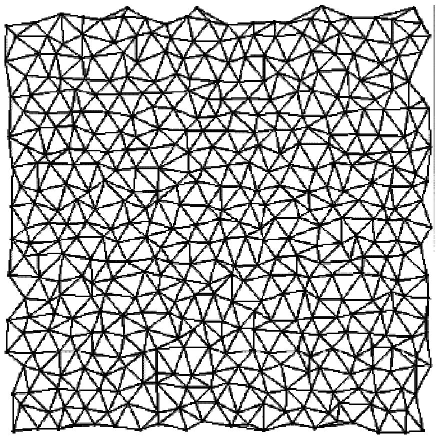
a)



b)



c)



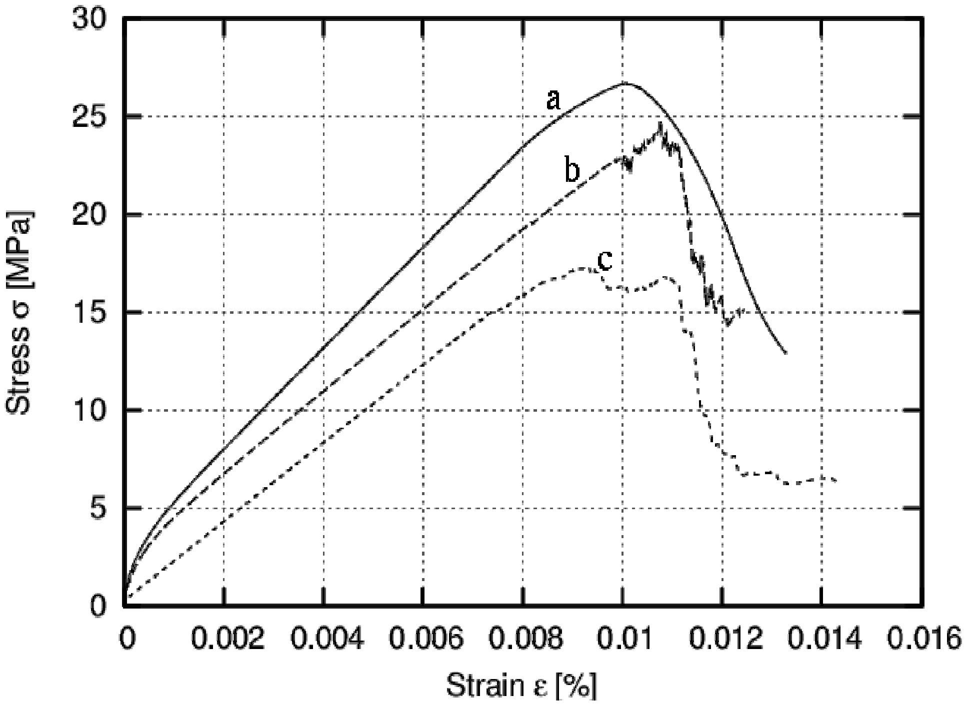
d)

Parameters used to describe the model

Group 1	**Stiffness parameters**	
	k_l	longitudinal stiffness [-]
	k_b	bending stiffness [-]
	k_t	torsional stiffness [-]
Group 2	**Fracture parameters**	
	ε_{min}	critical tensile strain [-]
Group 3	**Mesh generation parameters**	
	g	cell size [m]
	r_{max}	max beam length [m]*
	α	min angle between beams [rad]*
	s	mesh irregularity [m]

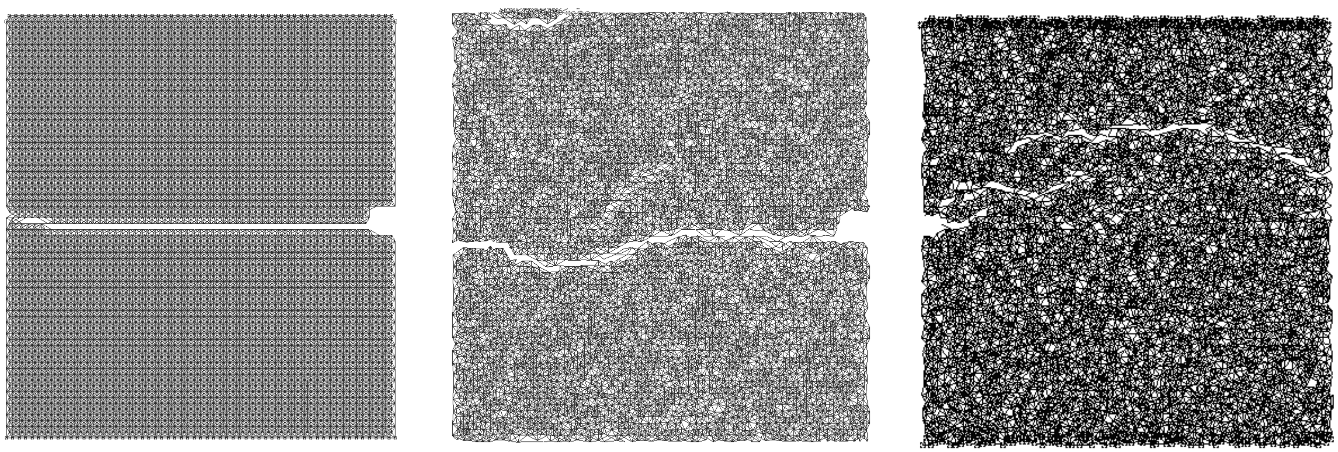
** parameters used only with non-Delaunay generation method*

Effect of mesh irregularity



Effect of mesh irregularity parameter ‘s’ on the stress-strain curve and crack pattern.

α=20°, r_{max}=2g, k_b / k_l=0.6
elements removed when ε_{min}=0.02%



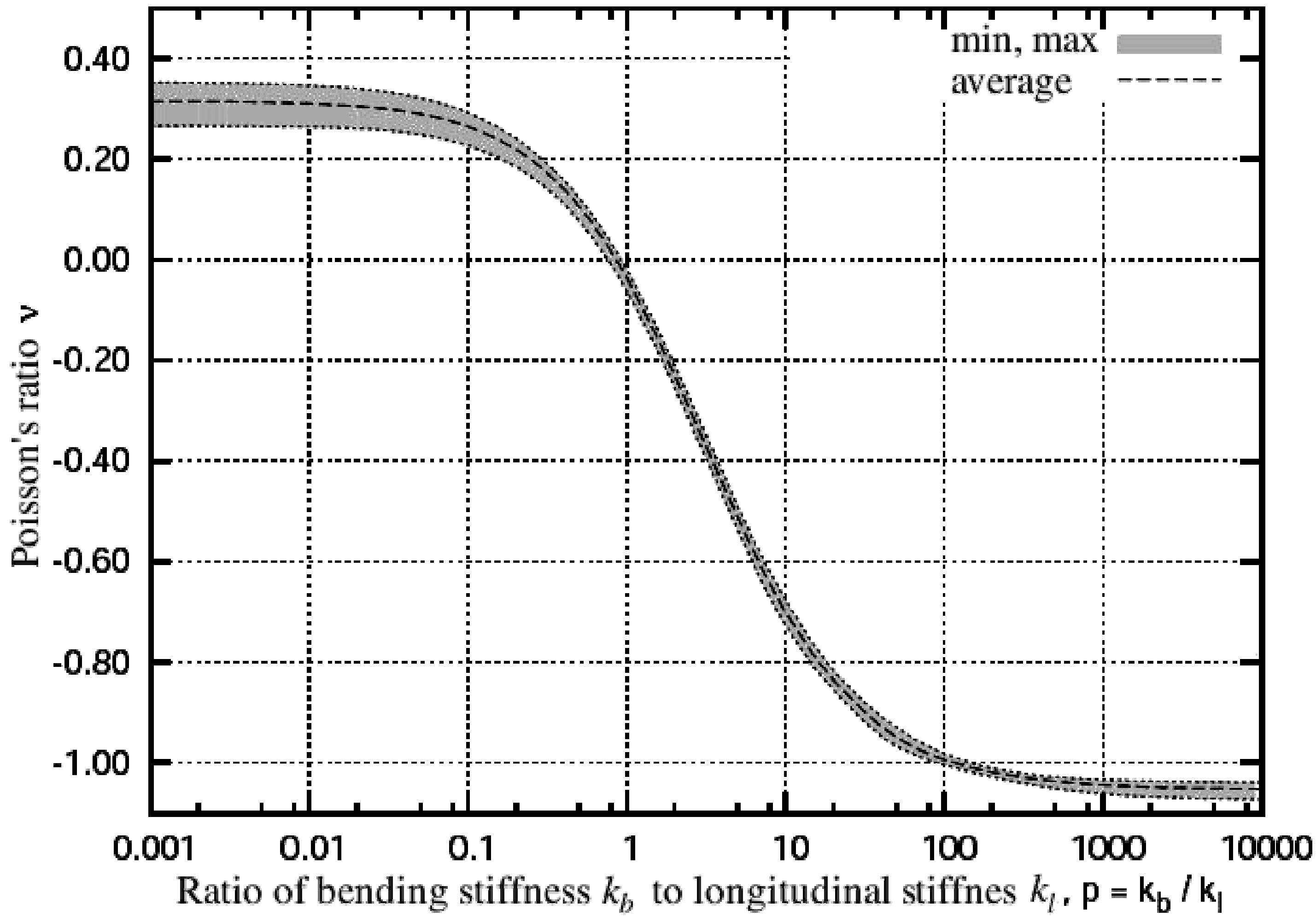
a) b) c)

a) s = 0

b) s = 0.3

c) s = 0.6

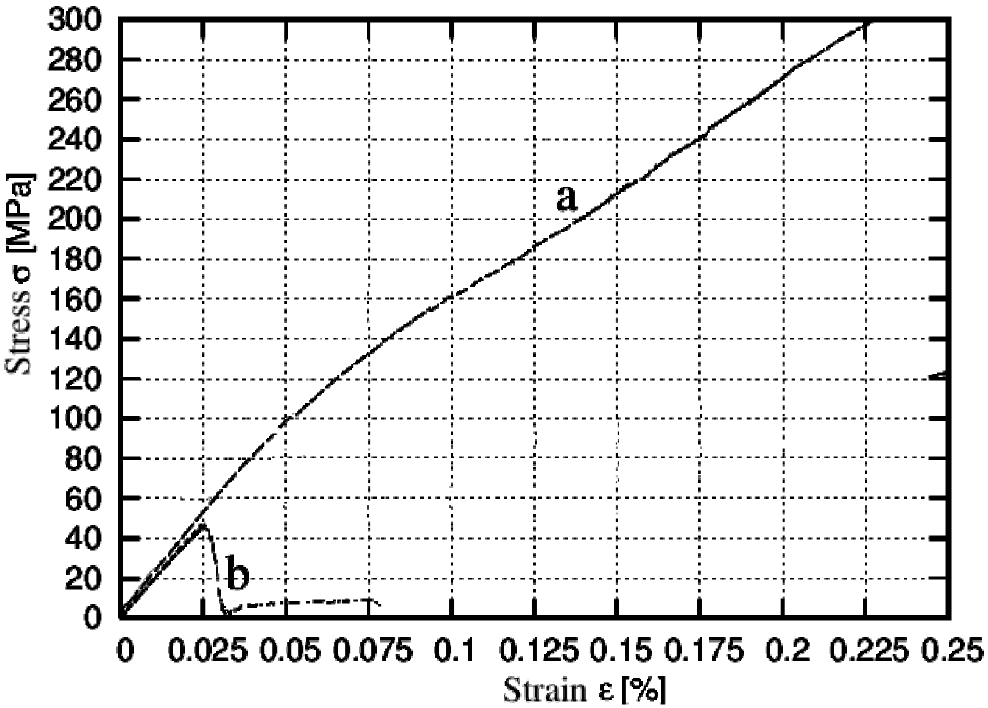
Influence of k_b on Poisson's ratio

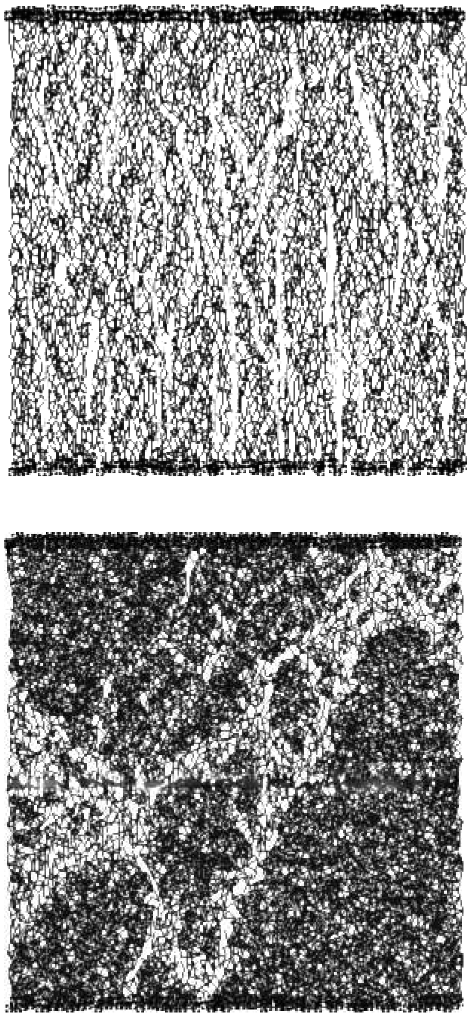


Ratio between bending and longitudinal stiffness $p=k_b/k_l$ determines the Poisson's ratio of the lattice mesh

Uniaxial compression (single phase)

Effect of stiffness ratio $p=k_b/k_l$ on stress-strain curve in uniaxial compression with smooth edges (elements removed when ε_{min}=0.02%)





a) p=0.3

b) p=0.001

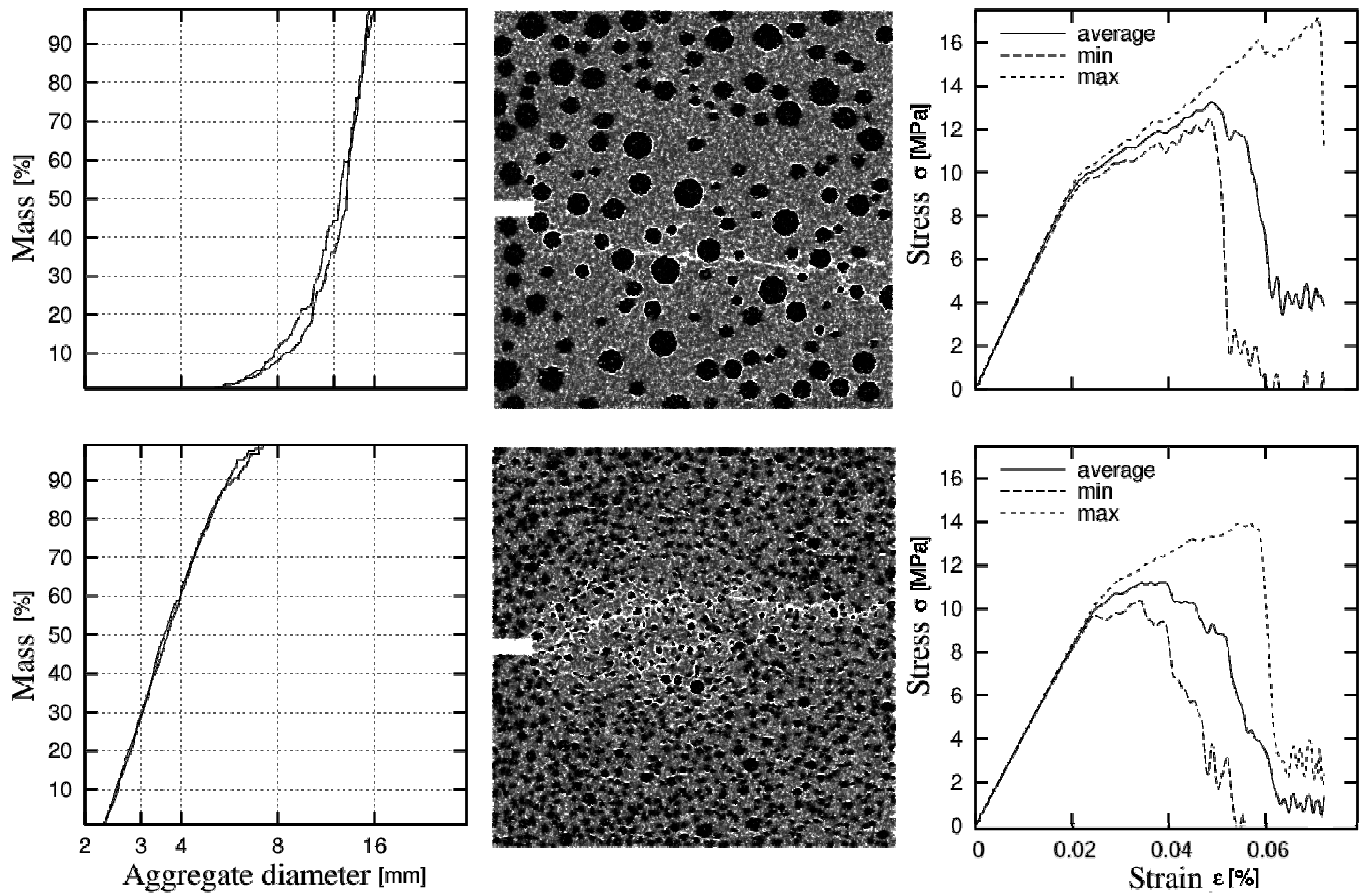
Effect of p on fracture pattern

Effect of aggregates – three phases 2D (25%)

Interfacial zone: k_l**=0.7,** k_b**=0.5,** ε_{min}**=0.005%,**

Cement matrix: k_l=1.0, k_b=0.7, ε_{min}=0.02%

Aggregate: k_l=3.0, k_b=2.1, ε_{min}=0.0133%



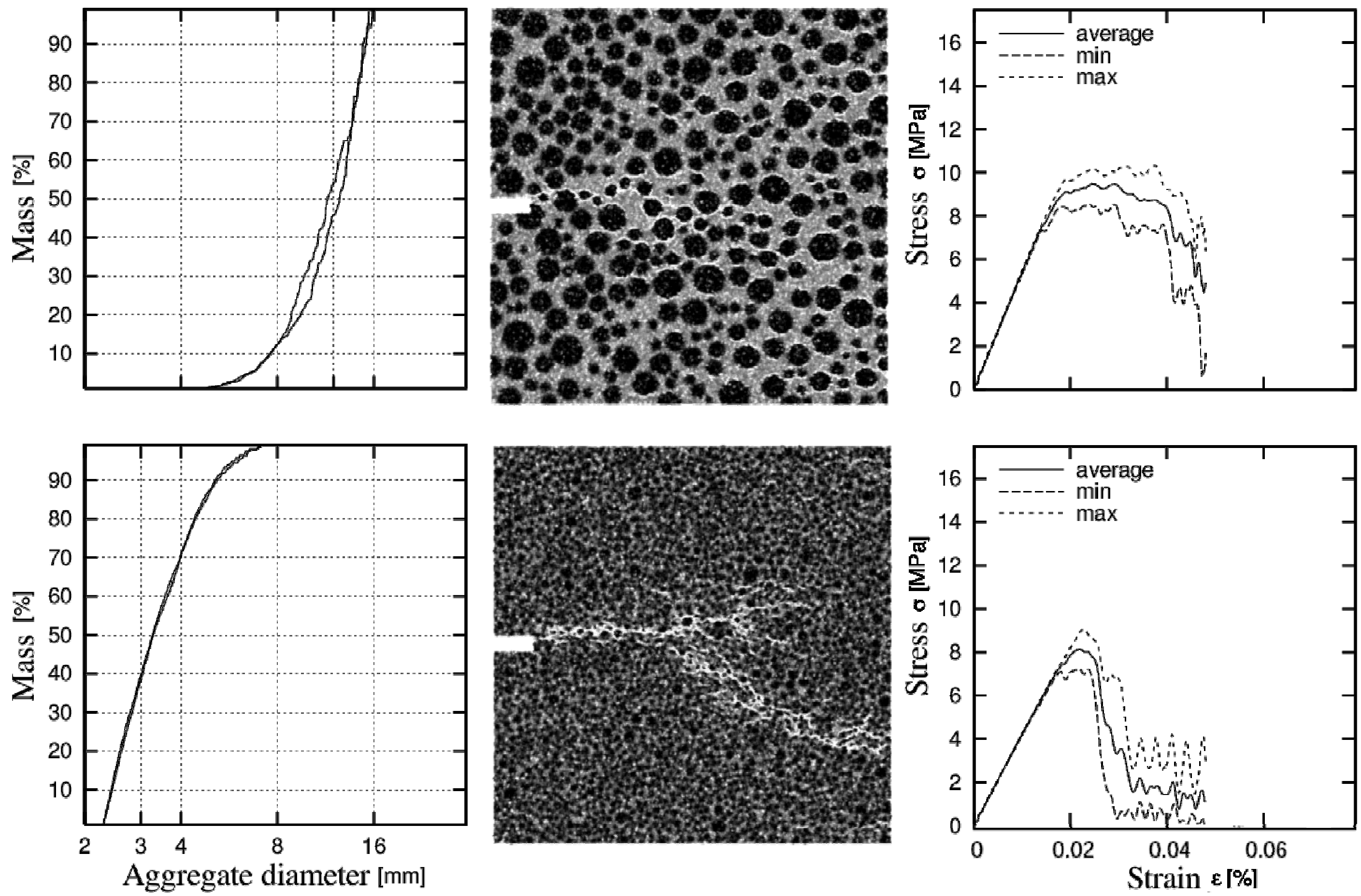
Uniaxial extension – 2 different sieve curves

Effect of aggregates – three phases 2D (50%)

Interfacial zone: k_l**=0.7,** k_b**=0.5,** ε_{min}**=0.005%,**

Cement matrix: k_l=1.0, k_b=0.7, ε_{min}=0.02%

Aggregate: k_l=3.0, k_b=2.1, ε_{min}=0.0133%



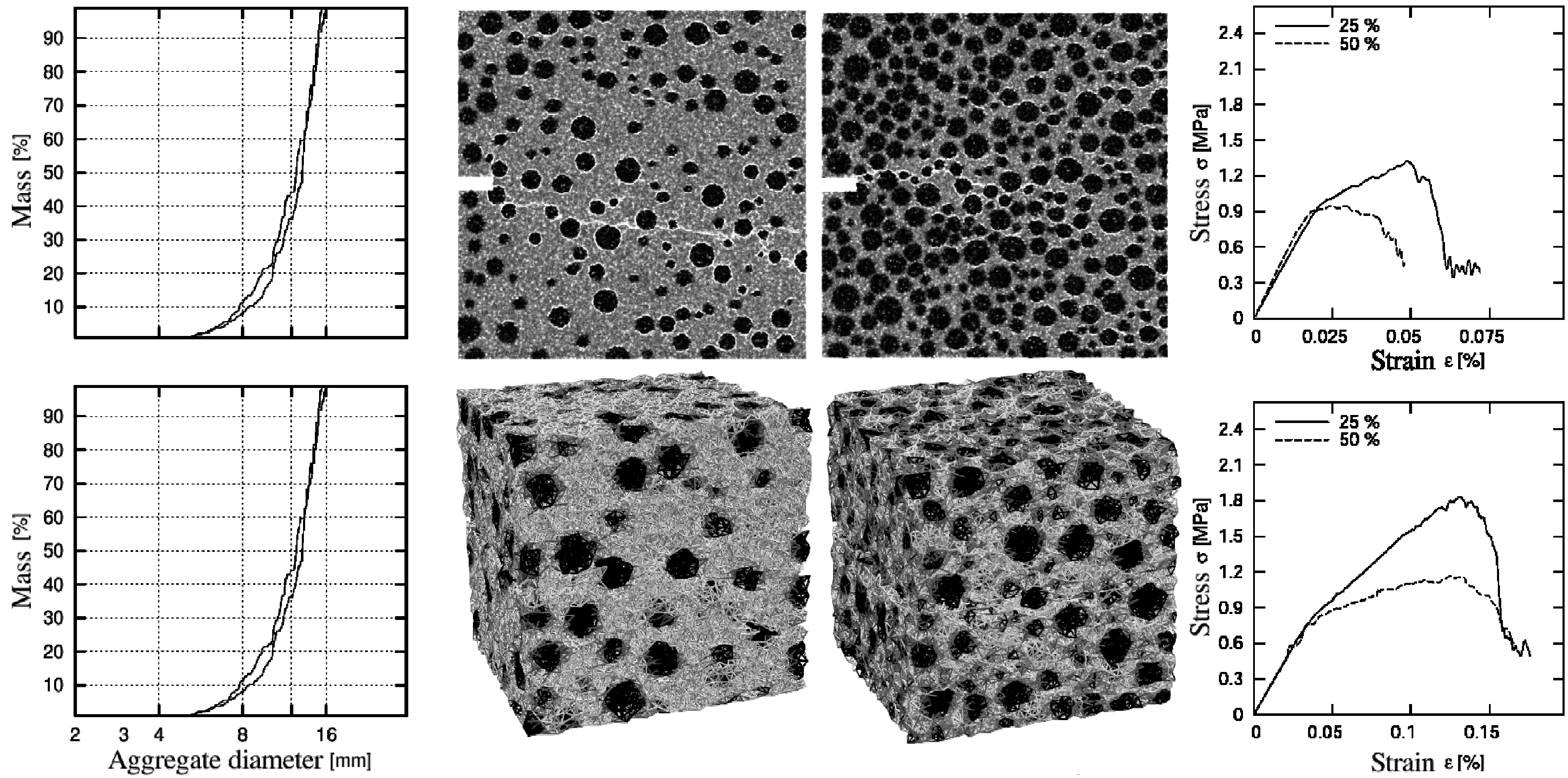
Uniaxial extension – 2 different sieve curves

Effect of aggregates – three phases 2D / 3D (25% / 50%)

Interfacial zone: k_l**=0.7,** k_b**=0.5,** ε_{min}**=0.005%,**

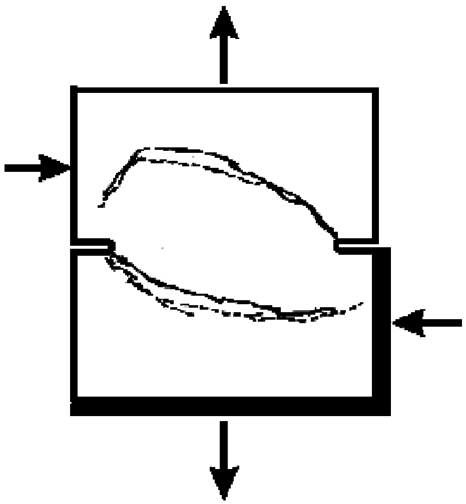
Cement matrix: k_l=1.0, k_b=0.7, ε_{min}=0.02%

Aggregate: k_l=3.0, k_b=2.1, ε_{min}=0.0133%

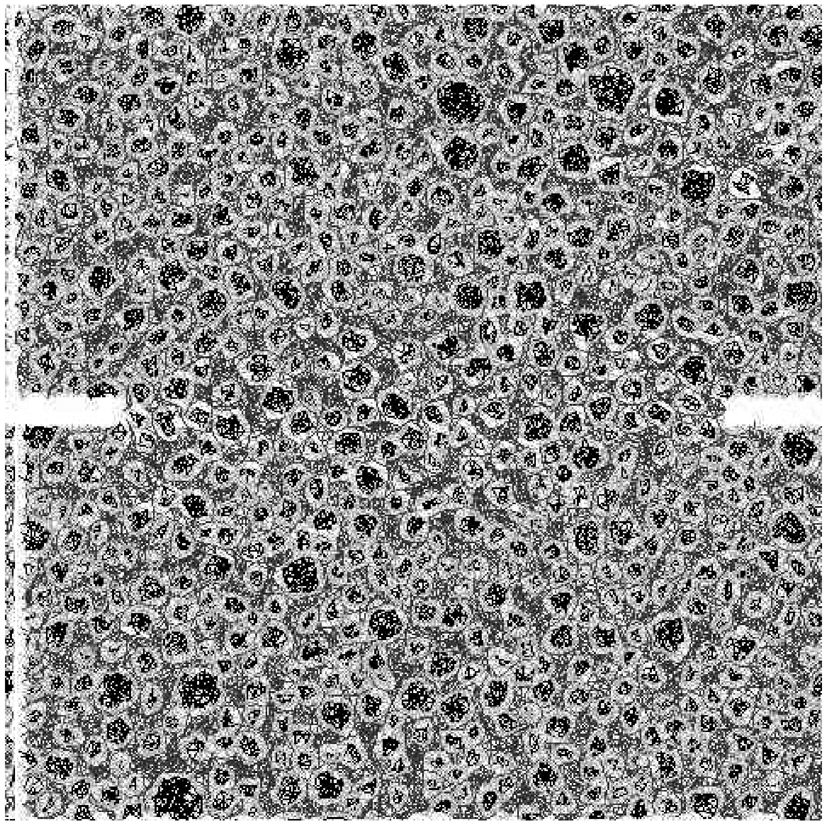


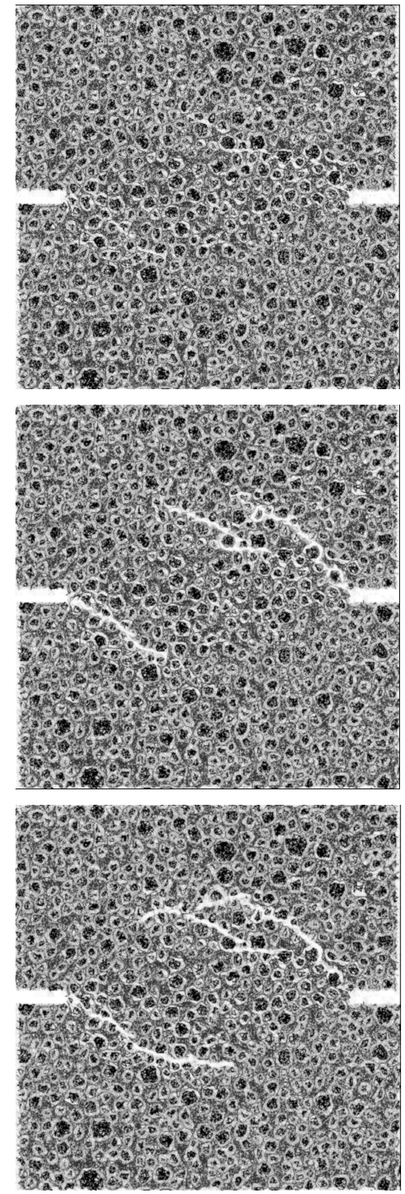
Uniaxial extension – 25% and 50% of aggregate volume/area

Nooru-Mohamed test (propagation of curved crack)
three phase material, 2D



Experimantal
fracture pattern
(Nooru-Mohamed 1992)

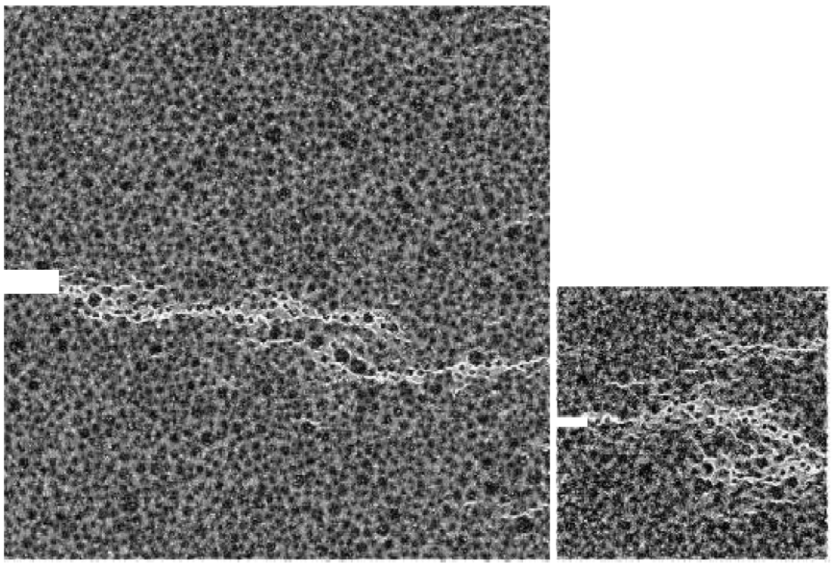




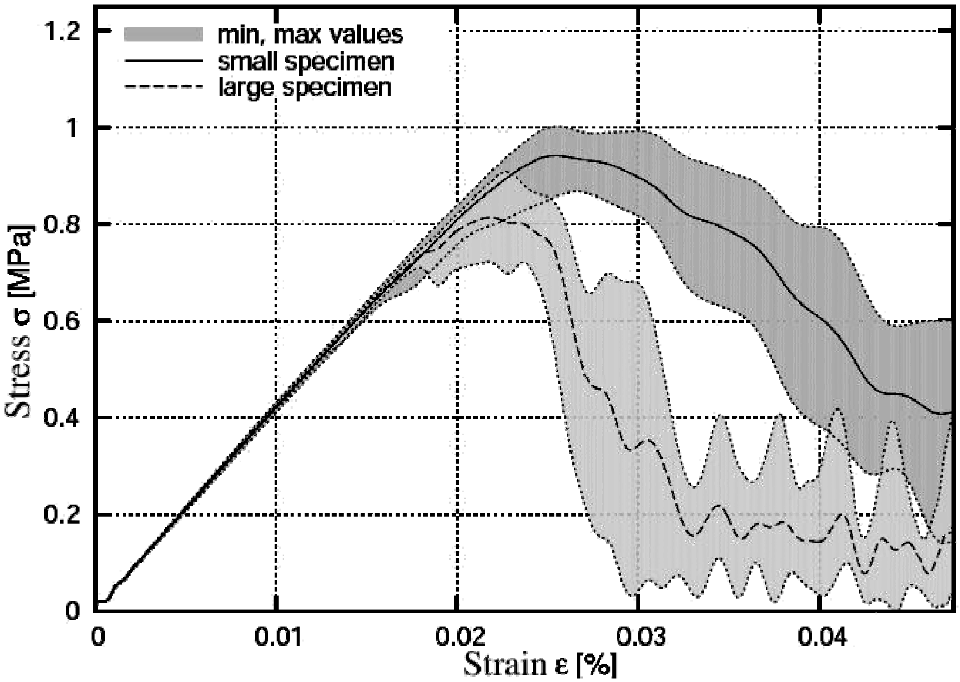
Crack propagation in numerical results

Size effect in numerical results (three phases 2D)

Interfacial zone:	$\boldsymbol{k_l=0.7, k_b=0.5,}$	$\boldsymbol{\varepsilon_{min}=0.005\%,}$
Cement matrix:	$k_l=1.0, k_b=0.7,$	$\varepsilon_{min}=0.02\%$
Aggregate:	$k_l=3.0, k_b=2.1,$	$\varepsilon_{min}=0.0133\%$



Fracture propagation:
small specimen: 10×10 cm^2
large specimen: 20×20 cm^2



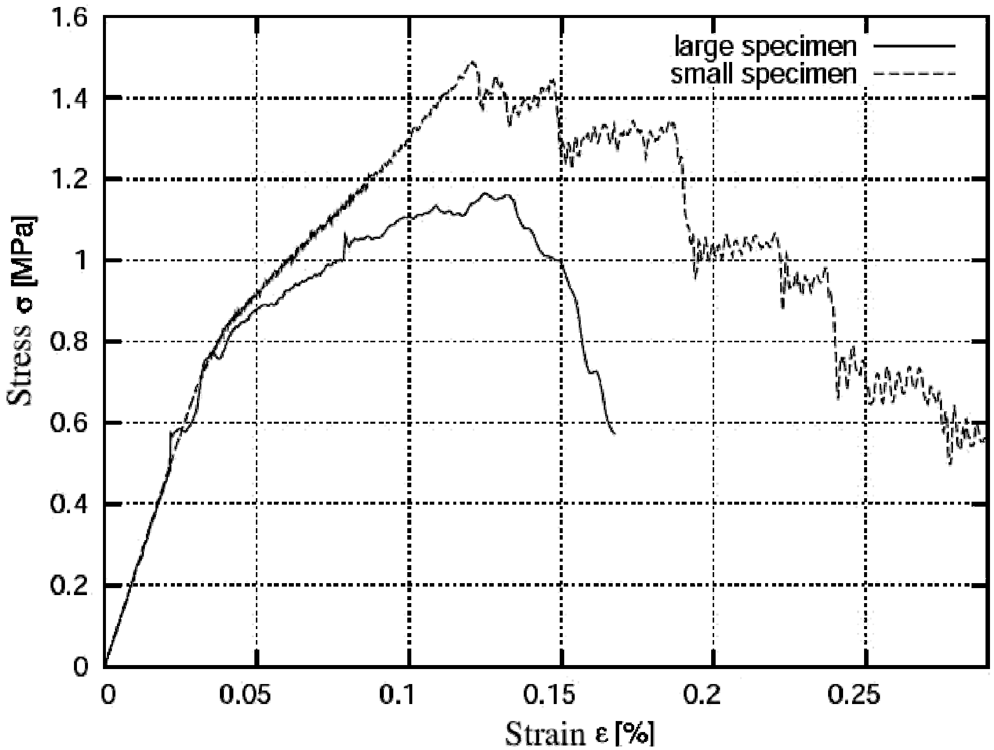
Stress-strain curve for **2D** specimens with different sizes subject to uniaxial extension with smooth edges

Size effect in numerical results (three phases 3D)

Interfacial zone: k_l**=0.7,** k_b**=0.5,** ε_{min}**=0.005%,**

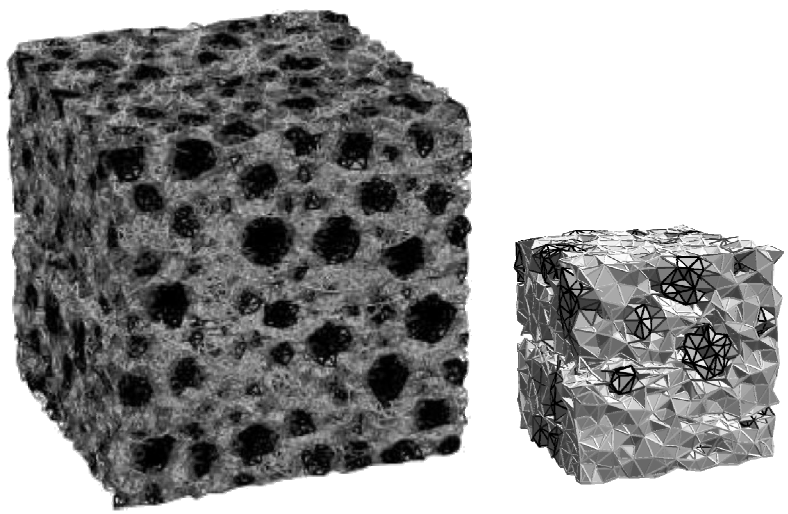
Cement matrix: k_l=1.0, k_b=0.7, ε_{min}=0.02%

Aggregate: k_l=3.0, k_b=2.1, ε_{min}=0.0133%



Stress-strain curve for **3D** specimens with different sizes subject to uniaxial extension with smooth edges

Assuming $k_t = k_b$



large specimen: $10 \times 10 \times 10$ cm^3

small specimen: $5 \times 5 \times 5$ cm^3

Conclusions

- Lattice model allows to study fracture propagation on the scale of cement matrix and aggregates.
- The peak-load decreases with increasing particle density.
- Ductility increases with increasing aggregate size and density.
- Small particle density leads to non-linearity in the pre-peak regime. High particle density leads to straight pre-peak stress-strain behaviour.